Combinations and Permutations

Updated
07/27/25

- Combinations: how many ways to arrange X things out of Y total things?
  Order *does not* matter
- Permutations: picking or doing things where order matters
  And, there are two types or permutations: with or without repetition

NOTE that these are different from Probability questions:

- *Probability: how many times does something happen out of a certain number of tries (or trials). The "something" that happens is the "favorable outcome". (Probability is covered in a separate resource document.)*

Examples of each type:

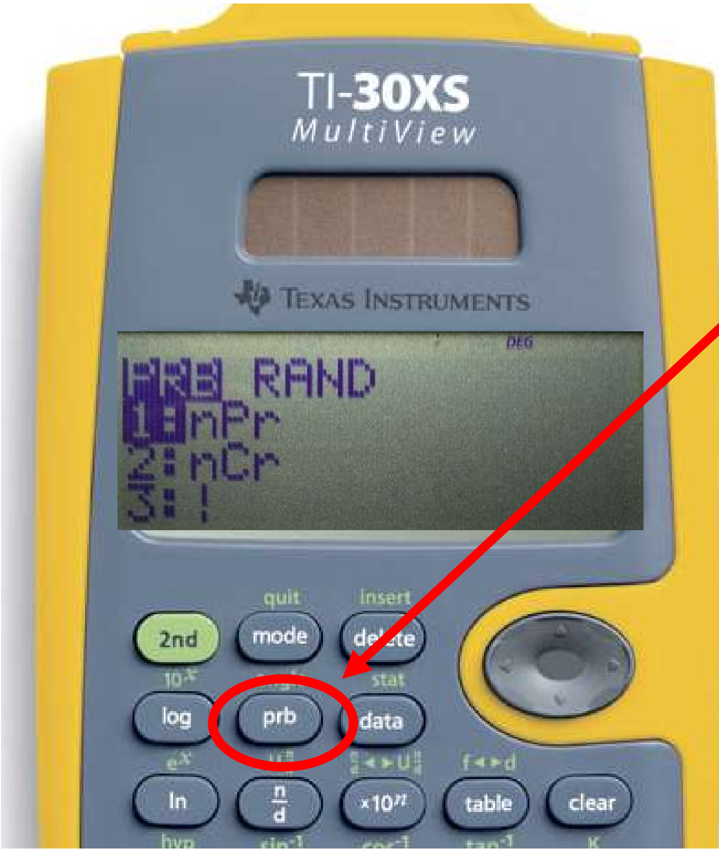
1. Combinations: Planting a flower bed. 4 flower types to choose from, but only want to use 3 types. How many combinations could be planted? *(Order of the flowers doesn't matter.)*
2. Permutation: A password has 4 characters that can be any letter of the alphabet (only small letters). How many unique passwords are possible? (repetition allowed) *(Order of the characters in the password matters.)*

For the combinations example, let's assume the flower types are represented by the letters A, B, C and D. The problem can be solved by creating a table and identifying the various combinations using just 3 types for each. Each line in the table represents a combination, so 4 combinations are possible.

This works, but is time consuming. We'll show how the Ti-30XS calculator can be used to *quickly come up with the answer*.

	A	B	C	D
ABC	x	x	x	
ACD	x		x	x
ABD	x	x		x
BCD		x	x	x

Learn to use the TI-30XS calculator "prb" button!

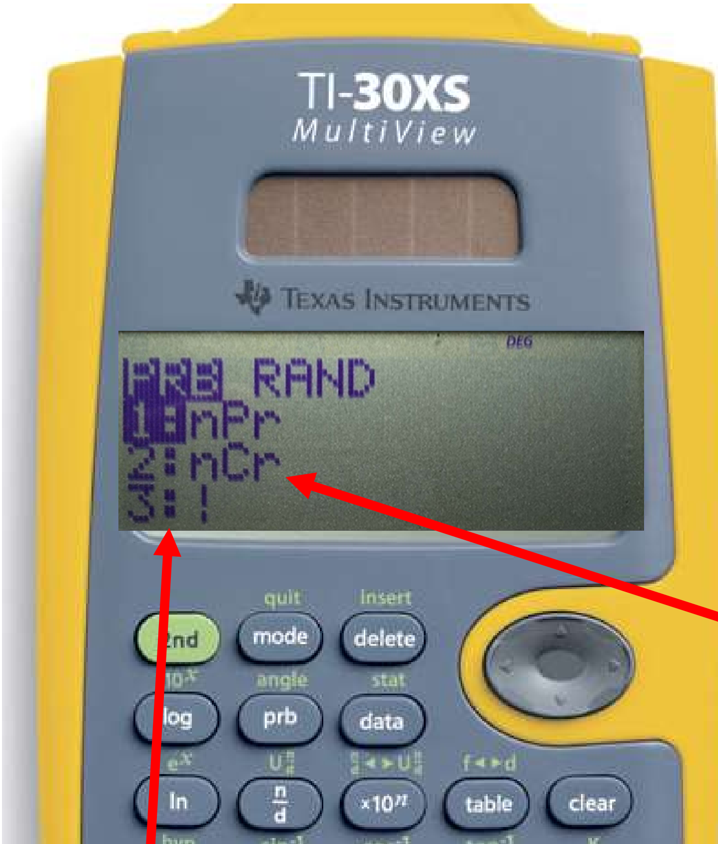


Use the "prb" key to access:
1: permutations calculations
2: combinations calculations
3: factorial calculations

Permutation Problem Steps:

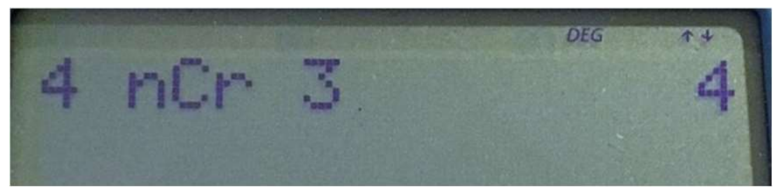
1. Enter the total number of things first (8 for this example).
2. Tap the "prb" key
3. First item, "1:nPr" highlighted, so tap "enter".
4. Enter the number of things being chosen (3 for this example).
5. Tap "enter" to see the answer

3 out of 8 things (order matters) –
Answer: 336 permutations

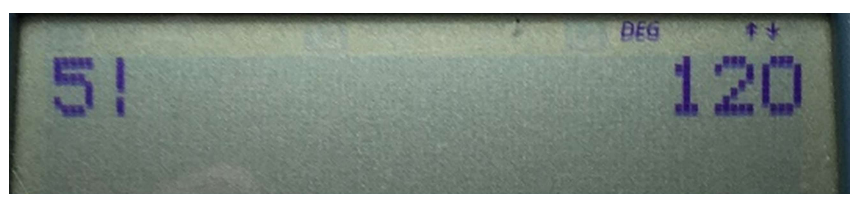


Example combination, 3 out of 4 things (order doesn't matter)

1. Enter 4
2. Scroll to 2nd menu item and tap "Enter"
3. Enter 3
4. Answer: 4 combinations when choosing 3 things out of 4 (much faster than creating a table!)



Example, 5! = 5 x 4 x 3 x 2 x 1 = 120
Enter 120, press "prb", scroll to 3rd menu option, hit "Enter"



Combinations with more than one type of thing: the fundamental counting approach

When the questions asks for the total number of combinations of multiple categories of things, *just multiply together the number of options in each category.*

Example 1: Francis has 3 pairs of slacks, 4 shirts and 2 hats. If an outfit is defined as one pair of slacks, one shirt and one hat, how many combinations are possible?

Answer: 3 x 4 x 2 = 24 combinations

Example 2: A restaurant has 8 appetizers, 12 entrees and 6 salads. If a meal is one appetizer, one entrée and one salad, how many different meals could be ordered?

Answer: 8 x 12 x 6 = 576 different meals

Permutations can be one of two types:

- With repetition
- Without repetition

Example permutation question with repetition:

A three-wheel "combination lock" has the numbers 0 thru 9 on each wheel. How many possible sequences of numbers are there? (And, for a lock, the order of the numbers matters – so this is a permutation, and for this example the numbers can be repeated.)

Answer: Each wheel has 10 numbers to choose from. So, there are 10 possibilities for the first number, 10 for the second, and 10 for the third.

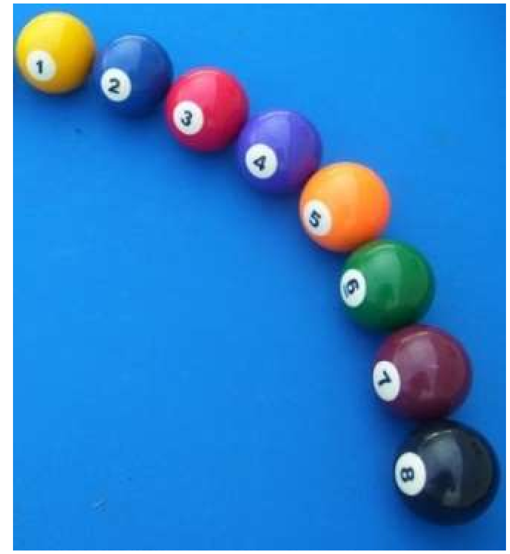
10 x 10 x 10 = 1,000 possible sequences

In the form of a general equation, let the number of possibilities be n, so the formula (with repetition) is:

$$n \bullet n \bullet n = n^3$$

Example permutation questions without repetition:

1. If you had eight numbered pool balls, in how many different sequences could they be arranged?



Answer: When you choose the first ball for a sequence, there are 8 to choose from. When you go to choose the second ball in a sequence, there are now 7 to choose from.

For the third choice there are now 6 to choose from, and the process goes on until only 1 is left. So, the calculation is:
8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = ***40,320 permutations (for sequences of all 8 pool balls)***

Note that this is 8! (8 factorial)

2 Suppose the question ask how many sequences there are if you were just choosing and arranging 3 balls at a time? The calculation is:

8 x 7 x 6 = **336 permutations (for sequences of 3 pool balls)**

Note: The equation for this is kind of complex – better to just remember the concept.

3 Consider, four students walking toward their school entrance. How many different ways could they arrange themselves in this side-by-side pattern? (Just give them numbers, 1 through 4. Each arrangement must be unique, so this is a permutation problem without repetition.)

The number of different arrangements is 4! = 4 • 3 • 2 • 1 = 24. There are 24 different arrangements, or permutations, of the four students walking side-by-side.

Practice Problems

1. Danny wants to take three video game cartridges with him on vacation, and he has five to choose from. How many possible combinations of three cartridges could he take along?

2. Ahmad is choosing four of his five employees for a special quality team. How many teams of four are possible?

3. A phone requires entry of a 5-digit sequence to unlock it. How many sequences are possible?

4. How many combinations of three are possible out of a group of six people? (Order does not matter.)

a. 720
b. 120
c. 180
d. 20

5. David has a small business with four employees. He’s going to attend a business seminar and he plans to take two of his employees along. How many combinations of employees could he take to the seminar?

a. 6
b. 4
c. 24
d. 10

6. At a local ice cream store, you can choose from 35 flavors of ice cream, 10 different toppings, and 2 containers — dish or cone. How many different combinations do you have to choose from?

7. How many unique 4-letter sequences can be formed from the word STUMPED?

8. Tara needs to choose an access code for her home security system that consists of four different digits using the digits 0 through 9. How many codes can be formed in this way? (Note that order matters. An access code of 3258 is very different from an access code of 8523.) There are ten digits to choose from: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. And all four digits in the code must be different.

9. Sebastian has 3 projects to complete at work today and he must decide in what order to complete them. How many possible orderings of three tasks are available to Sebastian?

10. How many different 5-letter arrangements are there of the letters in the word ”align”?

11. Ten students submitted their science fair exhibits in a contest. How many different sequences are possible for first, second and third place?

12. A chef is making deserts and has five types of frosting, three flavors of sprinkles and 4 different cookie cutter patterns. If each cookie will have just one type of frosting and one flavor of sprinkles, how many cookie combinations are possible?

Answers:

1. 10 (a combinations question: 3 of 5)
2. 5 (a combinations question: 4 of 5)
3. 100,000   (this is a permutation with repetition allowed)
4. d
5. a

6. 700 ice cream combinations. To answer this question, you multiply the three numbers with this reasoning: 2 container types times 10 different toppings times 35 flavors, so 2 x 10 x 35 = 700

7. 840 possible permutations. To answer this question, count the number of letters in STUMPED, and you get 7. For the first letter in the 4-letter permutation you have 7 to choose from, then 6, etc., and you get your answer: 7 x 6 x 5 x 4 = 840 (Or, you could use the calculator “prb” key and do a permutations calculation for 4 of 7.)

8. 5,040 different codes. There are 10 ways to choose the first digit in the access code. After that first digit is selected, it cannot be used again. That leaves 9 digits from which to select the second digit in the code. After the second digit is selected, it cannot be used again meaning that there are 8 options for the third digit. This leaves 7 digits for the fourth digit. Answer: 10 x 9 x 8 x 7 = 5,040 (Or, you could use the calculator “prb” key and do a permutations calculation for 4 of 10.)

9. 6 (a permutations question, without repetition: 3 x 2 x 1 = 6)
10. 120 (a permutations question, without repetition: 5 x 4 x 3 x 2 x 1 = 120. This is also 5!, which can be done with the calculator “prb” key.)
11. 720 (10 choices for first place x 9 choices for second place x 8 choices for third place = 720.)
12. 60 (5 frostings X 3 flavors X 4 cookie cutters)