Independent and Dependent Probability

08/08/25

Events or actions, as related to probability, can be independent or dependent.
At the time an independent event occurs, the event is not affected by any event that occurred previously. *A coin toss is a great example.*

A coin is not affected by any previous tosses. Each time it is tossed or flipped, there is an equal chance it will be a "heads" or "tails".
Suppose you call "heads". In the language of probability, the coin landing with the heads side up is called the favorable event or outcome. (It is "favorable" because that's what you want to happen, or it might be the thing you're trying to predict.)
To calculate the probability of an event, you divide the number of favorable outcomes (in this case one) by the number of possible outcomes (in this case two, because the coin could land in one of two ways).

$$P = \frac{1}{2}$$

So, there is a one out of two probability of heads. But, in the math language of probability, the probability will often by expressed as a percentage.

$$P = \frac{1}{2} = 0.50 = 50\%$$

In this case, there's a 50% chance that "heads" is the right call. This is independent: it doesn't matter what the previous flips were. It's a 50% probability each time.

Another example: Consider a pair of dice, each of which has 6 sides marked with dots. When you roll a single "die", each roll is an independent event. The probability of rolling a one is therefore:

$$P = \frac{1}{6} = 0.167 = 16.7\%$$

This is sometimes referred to as a "one in six" chance of something happening.

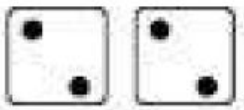
What if you're asked to calculate the probability of independent events occurring more than once? For example, what is the probability of flipping a coin twice and getting heads each time?
This is sometimes referred to as "combining" independent events. *For combined events, you just multiply the probabilities of the independent events*:

$$P = \frac{1}{2} x \frac{1}{2} = 0.25 = 25\%$$

So, the probability of flipping a coin twice and getting heads each time is 25%. (Or, a "one in four" chance since 25% is equal to 1/4 as a fraction.)

Similarly, the probability of rolling a pair of dice and getting a "two" on each one is:

$$P = \frac{1}{6} x \frac{1}{6} = \frac{1}{36} = 0.028 = 2.8\%$$



Note: Even though the dice are thrown at the same time, they roll independently – so this example is two independent events combined.)

A little more math language:

- Something that is impossible has a 0% chance of happening (or is a 0% probability).
- Events that have a 50% probability are sometimes referred to as having an even chance of occurring.
- Things that are certain to happen have a probability of 100%.

Dependent Events

Consider a deck of playing cards. A deck has 52 cards, and 4 of those are kings.
What are the chances of drawing a king on two successive draws? (And, the card drawn is NOT placed back in the deck.)
This is an example of dependent events. The action of drawing the first card alters the conditions related to the second card drawn. (So, one event depends on the other.)

The chance of drawing a king on the first draw: 4 possible kings out of 52 cards, or 4/52

But, assuming a king was removed from the deck on the first draw, the conditions for the second draw are 3 possible kings out of 51 cards, or 3/51

The probability of each of these two events are *multiplied together to get the overall probability of these independent events.*

$$\frac{4}{52} \text{ X } \frac{3}{51} = \frac{12}{2652} = \frac{1}{221}$$

So, the chance of getting 2 Kings is 1 in 221, or about 0.5%

Another dependent probability example considers 2 blue and 3 red marbles in a bag.

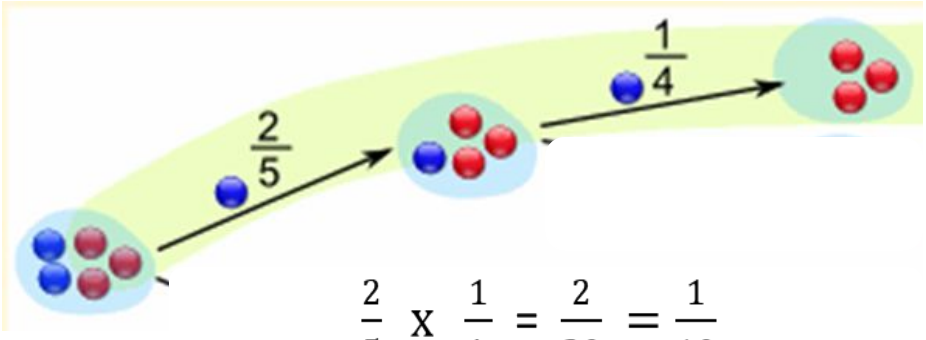
What is the probability of picking 2 blue marbles?

On the first pick, the odds (probability) of drawing a blue marble are 2/5

(2 blue marbles in the bag out of a total of 5 marbles)

For the second pick (assuming a blue marble was the first pick), the probability of drawing blue is now 1/4

(only 4 marbles remain, and only one is blue)



$$\frac{2}{5} \text{ X } \frac{1}{4} = \frac{2}{20} = \frac{1}{10}$$

So, there's a 1 in 10 chance of picking out 2 blue marbles in succession.

Sample Problems:

1. What's the probability of randomly picking a green lego part out of a group of 4 green and 8 black?
2. Katie tossed a coin 5 times and got heads each time. Stated as a percentage, what were the odds of that happening?
3. There are 12 red and 16 green candies in a bowl. If you pick one piece of candy and eat it, then pick another, what's the probability (as a percentage) that both were green?
4. You're playing a trick on a friend by throwing 4 extra puzzle pieces in the box of a 600-piece puzzle. What's the probability your friend will pick one of the 4 extras on the first pick? (State the answer as a __ in ___ probability.)

(Answers next page)

Answers:

1) There are 4 favorable outcomes (picking a green lego) out of 12 total possible outcomes (the total number of lego parts). So, favorable/total possible = 4/12, which simplifies to 1/3.
2) Each coin toss is a 1/2 probability of heads, so five in a row is 1/2 multiplied 5 times. That is equal to 1/32 or 0.03125. Multiply by 100 to convert to a percentage of 3.125%.
3) This is dependent probability. For the first pick, the probability of getting a green is 16/28 (total). Assuming a green was selected, the probability of a another green is 15/27 (total, since 1 green is gone – there are 15 greens left out of 27 total). So, the calculation: (16/28) * (15/27) = 0.317 which is 31.7%
4) Picking an extra piece with the first pick is a 4 out of 604 (total pieces) probability. But, 4/604 can be simplified to 1/151. And, the simplified probability can be stated as a 1 in 151 probability or chance.